Second letters from Sir John Herschel No. 5

I did not intend to have entered into the question with you as a matter of correspondence, but as I think you do not quite clearly perceive the point at issue, I will try to put it more distinctly than it would seem I have done.

M. Comte contends that he has shewn a numerical coincidence to subject between the periods of the planets as given by observation & as calculation on the nebular hypothesis.

Now the nebular hyp. makes two assumptions. 1st (assumption A) that the sun shrank in dimension, not arbitrarily, but according to certain determinate physical laws depending on radiation of heat & the constitution of its materials- its form, motion, & distribution of its strata at each instant following from those laws by purely dynamical considerations. (N B These laws I contend are utterly unknown, nor can I conceive any means whereby they can ever become known).

2d (Assumption B) That if the sun shrank in succession to the size of each planet & orbit is abandoned a portion of its exterior which became the planet, leaving it revolving in the orbit where it now is. & of course in the period it now has or then had —— & that for every planet in succession.

Assumption B is equivalent to that of the <???> of a planetary rotation between a superficial particle at the suns equator & the general attracting mass. In it assumes the centrifugal to be equal to the centripetal force, so that the superficial portions shall not press upon the interior body, & shall consequently be left behind as planetary molecules, as that body continues to shrink. If therefore we calculate from assumption B, the period (P) of the sun rotation at the equator where its radius (R) was equal to that of the orbit of a given planet, we necessarily arrive at a result ($P = \frac{2\pi}{\sqrt{M}} \cdot R^{\frac{3}{2}}$) identical with the derived period of the planet (which equation is what I <???> by Kepler's law) ; provided we suppose the sun at all times to

have been spherical. And if we take into consideration the oblateness, & put m for the mass which placed at the sun's center should attract

as the oblate mass M does —— then $P = \frac{2\pi}{\sqrt{M}} \cdot R^{\frac{3}{2}}$ will be the period:

which will agree neatly with the planetary periods (say within a 45^{th} part) if the ratio $\frac{\sqrt{m}}{\sqrt{M}}$ do not exceed, by more than $\frac{1}{45}$

To argue therefore from any exact or approximate coincidence between the results of such a calculation, & the actual planetary periods, is arguing in a vicious circle —— It is tautology.

Now this is what I contend M. Comte does in effect. The legitimate & logical course would be to shew that Assumption B is a necessary result from assumption A & that therefore the calculated periods flow, not from B as an arbitrary assumption, but from A. Now to do this on correct dynamical principles is not practicable by reason 1^{st} of our ignorance of the Physical laws alluded to (which must influence the result, as is easily shewn by trying a few cases) and 2^{d} by reason of the utter unmanageability of the analysis (we should have assumption upon assumption to make —— as to the law of coding —— of internal density of friction of gaseous shells reordering one within another in different times, & innumerable other such.)

M. Comte avowedly shies the difficulty & substitutes in place of a legitimate attack on the problem, a short cut.

This short cut consists in "combining" the Huygenian equation

Centrifugal force (C) = $\frac{V^2}{R}$ or which is the same thing $C = L\pi^2 \cdot \frac{R}{P^2}$

with "the law of Gravity".

The Law of Gravity is —— that the sun if a sphere attracts a superficial equatorial particle with a force = $\frac{M}{R^2}$ & if an oblate spheroid with a force = $\frac{m}{R^2}$ where m is an equivalent mass placed at the centre calculable when all the illiptricities & densities of the strata are known —— in other words it is the m above spoken of.

Therefore M. Comte's short cut consists in "combining" the two equations. C (= centrifugal force) $= \frac{V^2}{R} = 4\pi^2 \cdot \frac{R}{P^2}$ and G (= Gravity) $= \frac{m}{R^2}$

There are many ways in which two equations may be "combined."

1st. We may "combine" them by putting C = G. This is in fact arbitrarily & without any or <???> reason to make the assumption B —— & is the vicious circle I complain of.

2d. We may combine them in some other way —— but then C will not be = G; the centrifugal force will not balance gravity & the nebular hypothesis is at an end.

The following is a resumé of my argument

1. To calculate the periods from assumption B alone is tautological or a vicious circle. &

2. Assumption B cannot be legitimately deduced from assumption A, by reason. 1st of physical data being wanting, & 2dly by reason of the analytical difficulties of the problem.

3. M. Comte pretends to have deduced assumption B from A, or else the periods in question direct from A.

4. But his process amounts to assuming B, & can amount to nothing else, for centrifugal force can have nothing to do with the question except in so far as it goes to diminish the gravity —— & if it diminishes it to any point short of destroying it altogether, the nebular hypothesis is set aside.

If what is above said about m as distinct from M appears to puzzle the argument, it may be omitted without injuring its validity. Let the gravity to the sun's center be what it will. To assume that C = G is to assume the planetary rotation & to assume that C is not = G is to destroy the nebular hypothesis. It was incumbent on M. Comte to prove C = G through the medium of assumption A, not to assume it —— i.e. to prove his short cut by means of the very considerations it was designed to avoid.

I have left myself no room for entering into the other subject alluded to in your letter, & it will take me some little time to <???> my scattered ideas on the subject.

One word —— in reference to a point mentioned in your letter. You say "and if it can be shewn that the present rotation of the sun takes place in the same time in which it ought to take place supposing that proposition & the nebular hypothesis to be true, would not that be of considerable weight" &c ??

Doubtless it would —— but let us see how the fact stands .
The Sun actually revolves in $25\frac{1}{2}$ days.

A planet at the sun's surface would revolve in $2^{h}\ 46^{m}\ 48^{s}$.

I see no signs of a "numerical coincidence" here at least.

16 July 1845